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Runaway electrons in an air gap in the presence of a magnetic field

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Abstract. The divergence of the runaway electron flow generated in an air-filled discharge gap with a sharp conical cathode can be essentially reduced by applying a guiding axial magnetic field, which opens up prospects for the practical use of formed dense paraxial bunches of fast electrons. In the present work, we consider factors that determine the radial scale of the bunch. Our analysis shows that the main factor is the diffusion of electrons across the magnetic field lines due to collisions with gas molecules. Calculations of the dependence of the runaway electron beam radius on the magnitude of the applied magnetic field, taking this phenomenon into account, agree with the experimental data.
Keywords: runaway electrons, axial magnetic field, beam radius, diffusion across the magnetic field

1. Introduction

The study of runaway electrons (RAEs) in over-voltage gas gaps [1] is relevant because of their role in the breakdown development and prospects for applications for impact on different objects, media excitation, generation of electromagnetic radiation, etc. The main disadvantage of this source of fast electrons is the divergence of the RAE flow due to the character of the electric field distribution in the gap and particle scattering by gas molecules. For practical use, RAE flow should be focused in the paraxial region in order to increase the electric current density.

It has been recently shown [2] that the RAE flow divergence in the air-filled discharge gap with a conical cathode can be reduced by using a guiding axial magnetic field. This made it possible for the first time to form RAE bunches of comparable axial and radial sizes of several millimeters. The maximum RAE current density (0.3–0.65 kA/cm^2) is effectively controlled by the magnetic field (1.1–4.3 T).

In the present work, we consider factors that determine the beam radius and thus the maximum allowable values of the electric current density. As our analysis shows, the main factor is the diffusion of electrons across the magnetic field lines due to collisions with gas molecules; the diffusion coefficient satisfies the scaling law $D \sim 1/B^2$.

2. Experimental data and preliminary estimates

In experiments [2], a subnanosecond voltage pulse with amplitude of –155 kV was applied to the conical cathode (cone angle of 40^o and tip rounding radius of about 50 μm). RAEs were emitted at the front of the voltage pulse, when the potential difference U between the anode and cathode (30 mm gap) was ≈ 80 kV. This corresponds to the electric field strength at the cathode tip of above 10^3 kV/cm, which obviously exceeds the threshold value for electron runaway in atmospheric air of about 270 kV/cm [3]. A uniform magnetic field suppressing the RAE flow divergence was directed along the axis of the system. Three cases, B = 1.1, 2.1, 4.3 T, were investigated. It was found that the radius of the RAE beam is sensitive to the magnitude of the magnetic field: see data in Table 1. With an increase in B by 4 times, it decreased by more than 3 times from 2.6 to 0.8 mm.

Note that the decrease in the beam radius with increasing magnetic field is not directly related to the inverse dependence of the Larmor radius on B. Calculation of trajectories of electrons starting from the tip of the conical cathode in the vacuum approximation (i.e., without taking into account the interaction of electrons with gas molecules) gives a qualitatively correct character of the dependence on B, but predicts an order of magnitude lower values of the beam radius: from 300 to 80 μm (Table 1, right column). This means that the beam radius is mainly determined by the scattering of electrons by air molecules. In a vacuum, an electron moves oscillating around a certain magnetic field line. In a gas, collisions with gas particles lead to their displacement (diffusion) across the magnetic field: they begin to oscillate around new field lines.

Table 1. RAE beam (experimental and simulation results) for various magnetic fields

Magnetic field, T	Beam radius (experiment), mm	Beam radius (simulation in vacuum approximation), mm
1.1	2.6	0.30
2.1	1.4	0.15
4.3	0.8	0.08

3. RAE diffusion across the magnetic field

We have demonstrated above that, in order to interpret the results of experiments concerning the dependence of the RAE beam radius on the magnetic field, it is necessary to take into account the process of RAE diffusion across the magnetic field lines as a result of elastic scattering by gas molecules. This process is characterized by the momentum-transfer cross section σ_{em}, whose dependence on the electron energy ε can be approximated as $\sigma_{em} \approx \sigma_{app}(\varepsilon_{app}/\varepsilon)^2$ with ε_{app} = 1 keV and $\sigma_{app} = 10^{-17}$ cm^2 for $\varepsilon > \varepsilon_{app}$ [4]. The structure of this approximation corresponds to the Rutherford formula $\sigma_{em} \sim q_e^4/(m_e^2\varepsilon_0^2u^4)$, where ε_0 is the electric constant, u is the electron velocity, m_e is its mass, and q_e is the elementary charge (the dimensionless coefficient in this dependence is determined by the charge number of particles on which the electron is scattered). The electron-molecule collision frequency is given by $\nu_{em} = un\sigma_{em}$, where n is the gas concentration ($\approx 2.7\cdot10^{19}$ cm^{-3} under normal condition). The classical formula for electron diffusion [5] has the structure

$$D = ku^2\Omega^{-2}\nu_{em}\,, \tag{1}$$

where $\Omega = q_eB/m_e$ is the gyrofrequency, i.e., the scaling law $D \sim 1/B^2$ takes place (here k is the dimensionless coefficient of the order of unity). This dependence is realized if the condition $\Omega >> \nu_{em}$ is satisfied. Otherwise, the magnetic field does not affect the diffusion of electrons and it will be $D \sim u^2/\nu_{em}$. Let us estimate the frequency ratio $\beta = \Omega/\nu_{em}$ (the Hall parameter). The peculiarity of the process under consideration is that the energy of RAE and its velocity $u = (2\varepsilon/m_e)^{1/2}$ are continuously increasing. Assuming that $U \approx$ const, we take the average energy along the RAE trajectory of ≈40 keV. As a result, we get β = 100–350 at B = 1.1–4.3 T, i.e. always $\beta >> 1$, which gives grounds for using (1).

4. Dependence of the beam radius on the magnetic field

Let the function $\rho(r,t)$ give radial distribution of the RAE bunch charge. Its relation with the recorded in experiments charge $Q(r)$ that passes through collimator hole of radius r (see details in [2]) is given by

$$Q(r) = \int_0^r \rho(r,T_f)2\pi r dr, \qquad Q_{tot} = Q\big|_{r\to\infty}\,, \tag{2}$$

where Q_{tot} is the total charge of the RAE bunch, T_f is the RAE time-of-flight through the gap. The well-known solution that describes an axisymmetric diffusion process starting from a distribution in the form of δ–function, is as follows:

$$\rho(r,t) = \frac{Q_{tot}}{4\pi Dt}e^{-\frac{r^2}{4Dt}}\,, \tag{3}$$

It means that the RAE flow emitted at the moment $t = 0$ from, in fact, a single point (the tip rounding radius of the conical cathode is much smaller than the radius of the beam at the anode) will provide the following distribution at the moment $t = T_f$, when it reaches the anode:

$$Q(r) = Q_{tot} - Q_{tot} e^{-\frac{r^2}{4DT_f}}. \tag{4}$$

It is obtained by substituting (3) into (2).

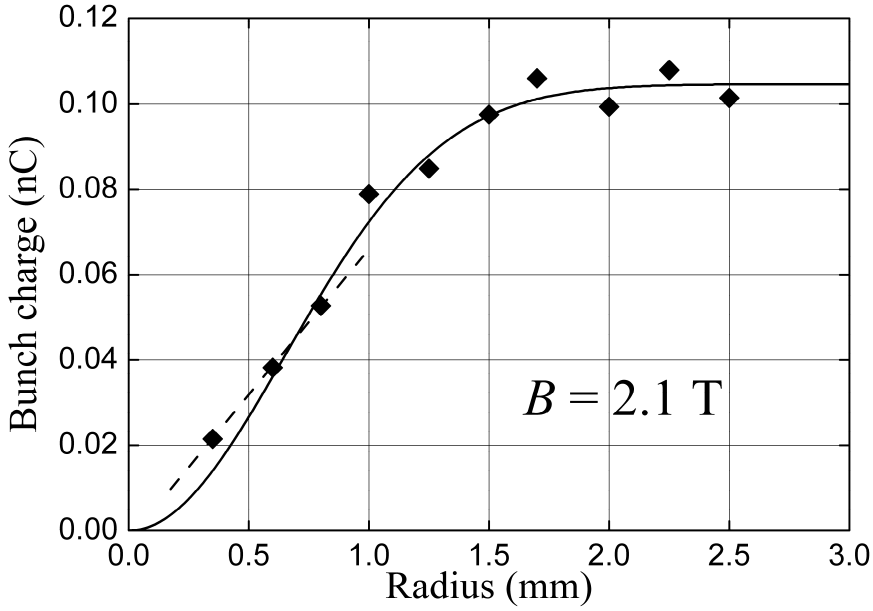

Fig.1. RAE bunch charge recorded at different collimator hole radii for $B = 2.1$ T.

Fig.1 demonstrates on the example of the case $B = 2.1$ T that the dependence (4) describes the experimental data quite well (solid line and rhombs, respectively). Here we have taken $Q_{tot} = 0.105$ nC and $(DT_f)^{1/2} = 0.46$ mm. A detailed comparison of the positions of the experimental points and the curve approximating them shows some differences in the region of relatively small r. Here the experimental dependence $Q(r)$ is close to linear one (dashed line in Fig.1), which formally ensures the divergence of the distribution $\rho(r)$ in the limit of small r: we have $\rho \sim 1/r$. Note that, of course, such a conclusion is not correct, since the radius of the collimator hole is always finite; its minimum value was 0.35 mm. The theoretical dependence (4) is smoother, $Q \sim r^2$, which provides a value of ρ close to the constant $Q_{tot}/(4\pi DT_f)$ at small r. Nevertheless, in general, the dependence (4) approximates the experimental data with acceptable accuracy, which makes it possible to use it to determine the diffusion coefficient D.

When using the approximation (4), it is convenient to choose the radius $r_{0.9}$ of the region through which 90 % of the charge passes as the beam radius. The condition $Q(r_{0.9}) = 0.9Q_{tot}$ gives $r_{0.9} \approx 3.03(DT_f)^{1/2}$. We take $T_f \approx 260$ ps (this value is obtained when the electron moves along the symmetry axis with taking into account the braking force in the gas in the Bethe approximation). We choose the diffusion coefficient in such a way that the experimental values of the radii coincide with $r_{0.9}$ for each value of the magnetic field. We find $D \approx 2800$ m^2/s for $B = 1.1$ T, 810 m^2/s for 2.1 T, and 260 m^2/s for 4.3 T. In Fig.2, these values are plotted (rhombs), and also the theoretical dependence (1) of D on B is given with $k = 4.1$ (solid curve). A fairly good agreement is seen, despite the qualitative nature of our consideration of the scattering of electrons by gas molecules (in particular, we did not take into account the fact that the velocity u and energy ε change along the RAE trajectory).

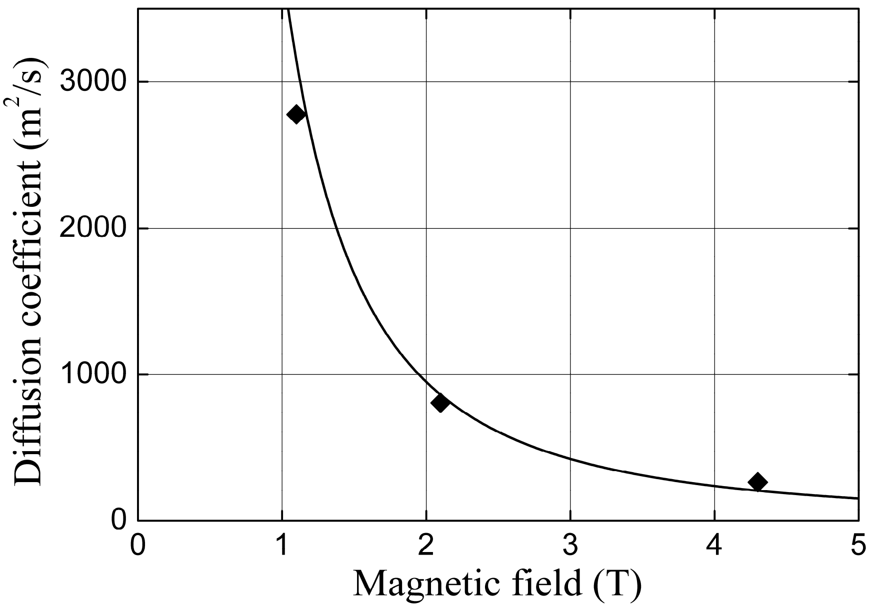

Fig.2. Calculated values of the diffusion coefficient D for B = 1.1, 2.1, 4.3 T (rhombs) and approximation $D \sim 1/B^2$.

5. Conclusion

In this paper, using the results of recent experimental work [2], we have established that the radial size of the magnetized RAE beam at the anode is determined by the process of diffusion of fast electrons across the magnetic field lines with the classical scaling $D \sim 1/B^2$. In this case, the relation $r_{0.9} \sim D^{1/2} \sim 1/B$ is valid for the beam radius. The obtained results will make it possible to predict the radial scale of the RAE flow, divergence of which is suppressed by a uniform guiding magnetic field. Note that the situation considered by us in [6], where tubular RAE flow was focused by an inhomogeneous magnetic field increasing along the electron trajectories, requires separate consideration. To calculate the spatial structure of the beam in this case, numerical simulation of the RAE propagation by the Monte Carlo method is required.

Acknowledgements

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