doi: 10.56761/EFRE2022.S3-P-033703

Nonlinear amplification of powerful terahertz pulses by electron bunches

*D.D. Krygina, Yu.S. Oparina, A.V. Savilov**

Institute of Applied Physics RAS, Nizhniy Novgorod, Russia
**savilov@ipfran.ru*

Abstract. There is a concept of the amplifier based on a principally non-linear effect of reflection of an electron bunch from the powerful wave pulse. This is effective mechanism of energy extraction by the wave from particles, when parameters of the electron bunch (initial energy spread, bunch length and emittance) haven't significant influence on the efficiency of the electron-wave interaction. Two schemes of possible realization of this process (non-resonant and resonant electron-wave interaction) are described.
Keywords: amplifiers, terahertz short pulse.

1. Introduction

The effect of electron acceleration in the fields of wave pulses has been well studied [1, 2]. In this work, we consider a possibility to reverse this problem and investigate the possibility of the amplification of a short powerful wave pulse by a photo-injector electron bunch due to the deceleration (reflection) of electrons in the terahertz (THz) frequency range. Obviously, in the case of the acceleration reversal the initial longitudinal velocities of electrons must exceed the group velocity of the pulse. Thus, this process should be realized in a media with the dispersion, so that the group velocity of the wave pulse is small enough. We consider the electron-wave interaction in the smooth cylindrical waveguide (Fig.1), as an example. The electrons catch up with the pulse, and it reflects them, so that their final velocities become less than the group velocity of the wave packet. As a result, a significant part of kinetic electron energy is passed to the wave.

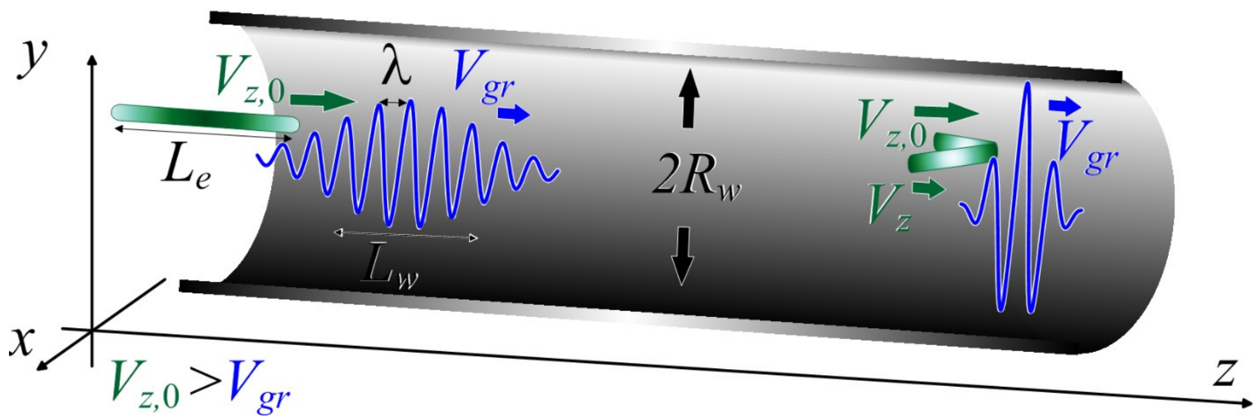

Fig.1 "Fast" particles move in a waveguide and catch up with the pulse. They are decelerated to velocities, which is lower, than the group velocity of the wave packet in the waveguide, and therefore amplify the radiation.

We describe two schemes of realization of this process. The first one is the non-resonant amplification of pulses due to braking in a waveguide under effect of the non-resonant pondermotive force (Fig.2a). In this case, particles are decelerated under the pondermotive force. Second one is the resonance amplification of weak pulses is described (Fig.2bc); here, the undulator resonance, as an example, is used to provide an efficient electron-wave interaction.

2. Non-resonant reflection of particles from high-power wave pulses

Consider a wave packet in a waveguide propagating with the group velocity V_{gr}. A bunch overtakes wave packet and is "reflected" by the wave field: particles velocities decrease to the ones, which is smaller than V_{gr}. If velocities of particles are various, particles are decelerated in different points. Obviously, particles with maximal energies propagates further. These particles should not propagate pass the point of the maximal pulse amplitude. Under certain conditions, it is possible to provide reflection of all particles. Let's analyze the particles dynamics in the approximation of the unchanged profile and amplitude of the wave pulse.

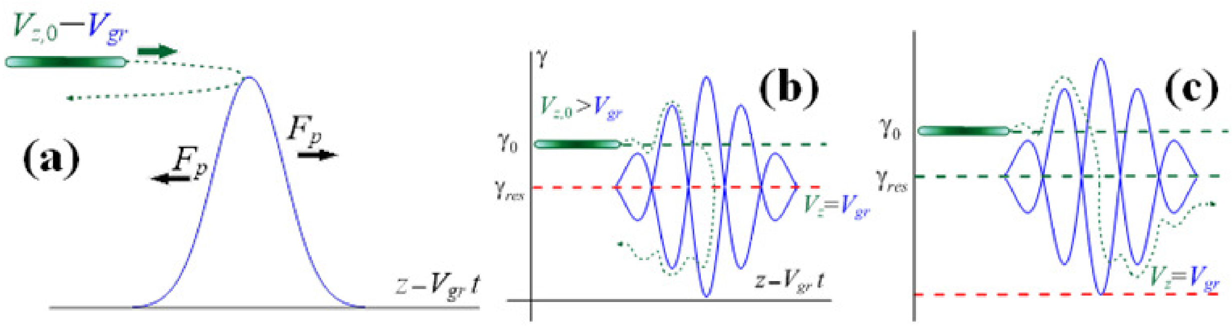

Fig.2. The deceleration of "fast" particles under effect of the non-resonant pondermotive force (a). Phase planes for particles in the case of "optimal" resonance energy level $\gamma_{res} = (1-\beta_{gr}^2)^{0.5}$ (b), and for $\gamma_{res} > (1-\beta_{gr}^2)^{0.5}$. The particles oscillate in the wave field before being captured and decelerated to velocities that are lower than the group velocity of the wave pulse (c). The optimal level of the resonance level is also shown.

Equation of electron motion in vector form:

$$\frac{d\boldsymbol{p}}{dt} = -\frac{e}{c}\left[\boldsymbol{V}, \boldsymbol{B}_w\right] - e\boldsymbol{E}_w . \tag{1}$$

Here, $\boldsymbol{p}$ – electron momentum, $\boldsymbol{V}$ – its velocity; $\boldsymbol{B}_w$, $\boldsymbol{E}_w$ – magnetic and electric fields of the wave, accordingly. Let 's move on to normalized quantities $p_+ = p_x + ip_y$ – normalized transverse complex momentum, $p_{x,y} = \gamma\beta_{x,y}$, $\beta_{x,y}$ – normalized electron velocity components, γ – relativistic Lorentz-factor; $p_z = \gamma\beta_z$ – the longitudinal normalized pulse. The particles interact with a wave pulse whose normalized vector potential $a_+ = a_x + ia_y = A_+/500\text{kV} = a(z, t)\exp(i\omega t - ihz)$, $a(z,t)$ – function describing the longitudinal structure of the field; ω, k – frequency and wavenumber (h is the longitudinal wavenumber). Consider (1) in the variables

$$\tau = \omega t\, , \zeta = k\left(z - V_{gr}t\right).$$

V_{gr} – group velocity of pulse. In the considered approximation $a(z, t) = a(\zeta)$.

In these variables, the equations for dimensionless momentum projections are written as follows:

$$\frac{dp_+}{d\zeta} = \frac{da_+}{d\zeta}, \tag{2}$$

$$\frac{dp_z}{d\zeta} = -Re\left[\frac{p_+^{\,*}}{\gamma\left(\beta_z - \beta_{gr}\right)}\frac{da_+}{d\zeta}\right]. \tag{3}$$

Change in energy is determined by the next equation:

$$\tau = \omega t\, \frac{d\gamma}{d\zeta} = -Re\left[\frac{\beta_{gr} p_+^{\,*}}{\gamma(\beta_z - \beta_{gr})}\frac{da_+}{d\zeta}\right]. \tag{4}$$

The change in the transverse moment is determined by the change in the wave field (we believe that initially the particles have only the longitudinal component of the velocity):

$$p_+ = a_+ ,\ p_{+,0} = 0 .$$

Multiply equation (4) by β_{gr}r and subtract from equation (3). We get the following result:

$$\frac{d\left(p_z - \beta_{gr}\gamma\right)^2}{d\zeta} = -\left(1-\beta_{gr}{}^2\right)\frac{d\left|a_+\right|^2}{d\zeta} . \tag{5}$$

For an arbitrary-shaped pulse, we obtain the result:

$$\left(p_z - \beta_{gr}\gamma\right)^2 = \left(p_{z,0} - \beta_{gr}\gamma_0\right)^2 - \left(1-\beta_{gr}{}^2\right)a\left(\zeta\right)^2 . \tag{6}$$

The change in the difference between the longitudinal velocity of the particles and the group velocity of the pulse,

$$\beta_z - \beta_{gr} = \frac{\sqrt{\left(p_{z,0} - \beta_{\mathrm{gr}}\gamma_0\right)^2 - \left(1-\beta_{gr}{}^2\right)a\left(\zeta\right)^2}}{\gamma} . \tag{7}$$

We are interested in the deceleration of particles to velocities less than the group velocity of the pulse, this requirement can be expressed by the following equation:

$$\left(p_{z,0} - \beta_{gr}\gamma_0\right)^2 = \left(1-\beta_{gr}{}^2\right)a\left(\zeta\right)^2 . \tag{8}$$

For the amplitude in the point of reflection it is rewritten as follows

$$a\left(\zeta\right) = \frac{\gamma_0\left(\beta_{z,0} - \beta_{gr}\right)}{\sqrt{1-\beta_{gr}{}^2}} . \tag{9}$$

The estimations for the electron energy change give:

$$\gamma - \gamma_0 = -\frac{\beta_{gr}\gamma_0\left(\beta_{z,0} - \beta_{gr}\right)}{\left(1-\beta_{gr}{}^2\right)} \propto a\left(\zeta\right) . \tag{10}$$

This means, that the efficiency is proportional to the amplitude in the point of reflection.

The important advantage of this method of the amplification is the possibility to use electrons with the initial energies up to 1 MeV for the amplification in the THz frequency range. As an example, we considered the amplification of a wave pulse, the initial duration of which is approximately 0.2 ns (it accords to 60λ, wavelength is 1 mm) by an electron beam, the initial particle energy is 350 keV, its current is 1 kA and the length is 0.1 ns. The interaction takes place in a waveguide with a diameter of 1 mm. According to the estimation (9), the peak power of the pulse must exceed 36 MW (it accords the amplitude 0.1) to ensure the reflection of all particles. The results of numerical simulations in the Fig.3 confirm this statement. There are dynamics of particles energies in relation to the pulse center and the changes of its amplitude during the interaction process. The interaction becomes considerable close to the point $\zeta = -L_{w,0}/2$ and after it. Particles in the front of the bunch are decelerated, that leads to the local amplitude increase. At the distance of three and half thousand wavelength all particles are reflected by the pulse profile and the pulse amplitude reaches the maximum value. In this example the peak power amplification by factor four takes place. After that the efficiency $\eta[\%] = \langle\gamma - \gamma_0\rangle/(\gamma_0 - 1)$ (here, $\langle\ldots\rangle$ means particles averaging) goes to the saturation at the level of 12%. Note, the duration of the amplified segment is about 15λ.

Thereby, we have an opportunity to get shorter, in comparison with initial duration, pulses with the sufficiently high peak power.

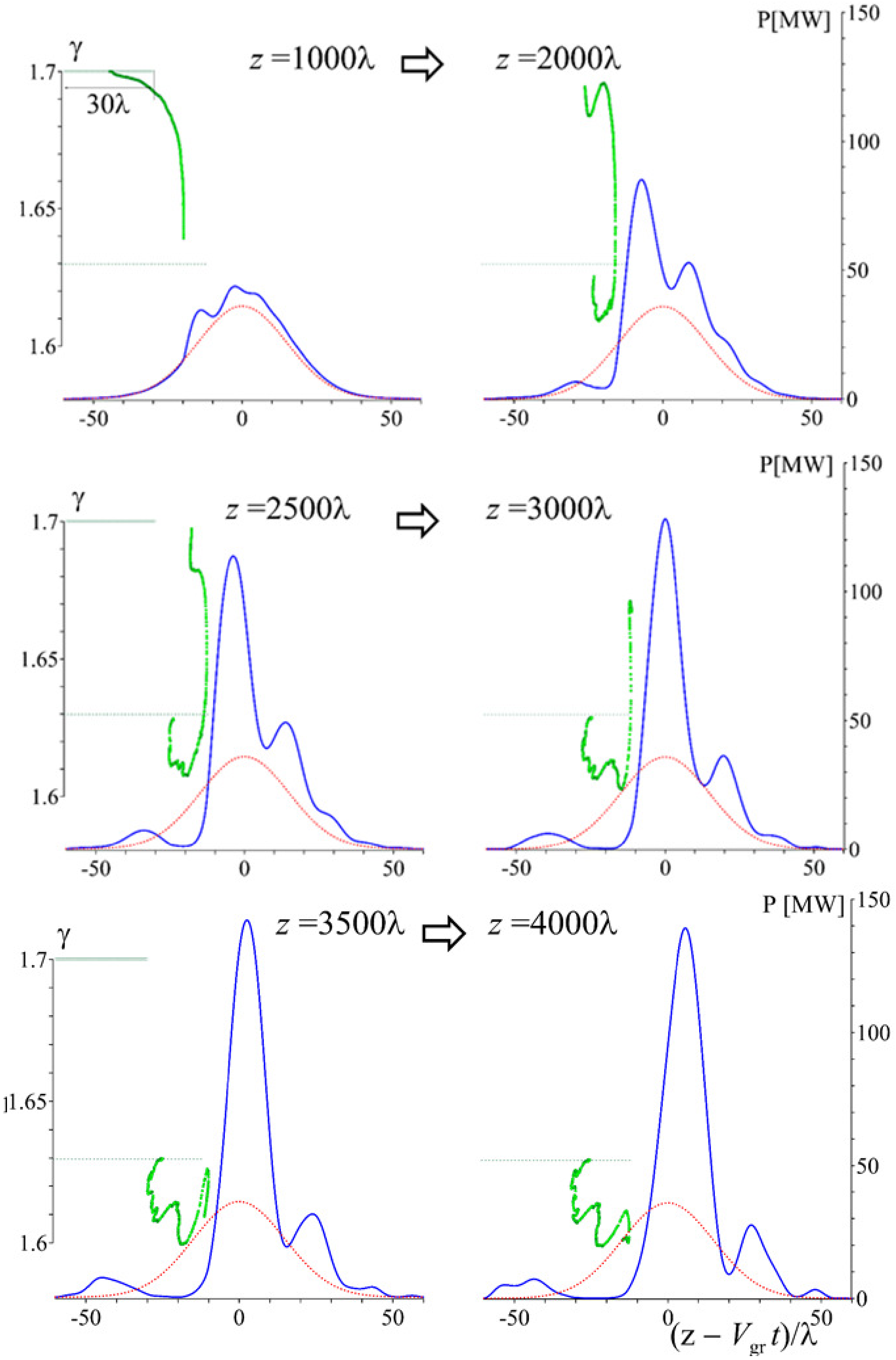

Fig.3. Dynamics of the distribution energies of particles in the process of their reflection from a wave pulse, and the amplification process (initial bunch length 30λ). Initial positions of particles, the energy level corresponding to the equality of particles velocities to the wave pulse group velocity and the initial pulse form (red dashed curve) are shown.

3. Reflection of particles in the case of the resonant electron-wave interaction

We include the undulator in the system (see Fig.2b). In this case, the change in the electrons energies is assessed by the following ratio [3]:

$$\langle \Delta\gamma \rangle \propto -\sqrt{a(\zeta)}\,. \tag{11}$$

So, it is possible to provide the effective amplification in the case of relatively small wave amplitudes. However, unlike the previous case, the bunches with initial energies at the level of 3–6 MeV are more appropriate (in the case of energy 350 keV the undulator period must be close to the wavelength). There is the resonance energy level, according to the resonance of electrons with the amplified wave, in the system. The optimal position of energy level accords to the $\gamma_{res} = \gamma_0 - (a(\zeta))^{0.5}/2$, and the group velocity of the wave packet accords to the resonance energy $\beta_{gr} = (1-\gamma_{res}^{-2})^{0.5}$ (see Fig.2b). In this case, almost part of electrons is decelerated to velocities, which don't exceed he group velocity of the wave. Results of numerical simulations (which are based on quasi-analytical models developed in our previous works [4, 5]), which are carried out for the gaussian longitudinal field distribution (the maximal normalized amplitude 0.01), confirm this statement (Fig.4). There is the dynamics of particles energies (initial energies $\gamma = 6.98 \pm 0.02$) as a function of theirs position relative the pulse center for two values of the group velocity. The undulator is circular polarized, its parameter 0.7. First, when the resonance energy is optimal $\gamma_{res} = (1-\beta_{gr}^2)^{0.5}$ (Fig.2b), and all particles are reflected (Fig.4a). In the second considered case of the resonance energy, which exceeds the value of energy according to the group velocity, $\gamma_{res} > (1-\beta_{gr}^2)^{0.5}$ (Fig.2c), it is realized by adjusting of the undulator period.

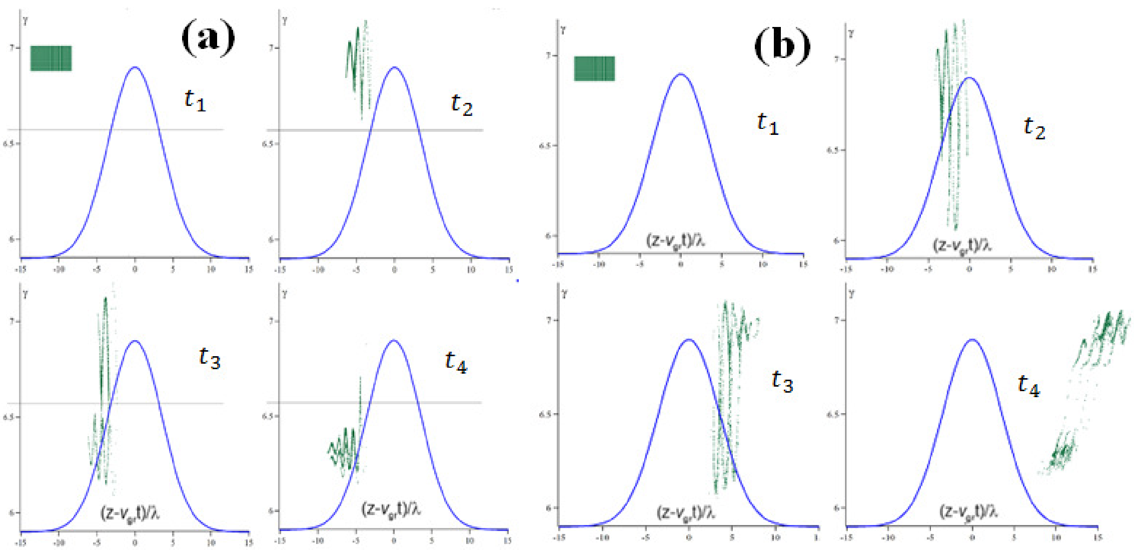

Fig.4. Dynamics of the distribution energies of particles in the process of their reflection from a wave pulse with an initial amplitude $\gamma_{res} = (1-\beta_{gr}^2)^{0.5}$ (a) and $\gamma_{res} > (1-\beta_{gr}^2)^{0.5}$ (b).

Focus on the optimal case shown in the Fig.4a. Particles oscillated in the combination wave field, some of them are trapped, the others propagates in the field maximum region, where they are decelerated to the sufficiently small velocities are "unfolded" relative to the pulse. In the considered case, average loses of particles energies are approximately 10%, that is in a good agreement with the estimation (11). The initial amplitude of the radiation power is 7.5 MW. Thus, for example, if the frequency of radiation is 300 GHz, the undulator period is 23.8 mm, and the waveguide diameter is 4 mm, the expected amplification by a bunch with total charge 0.1 nC will be approximately in three-four times.

It is interesting to note, that in the case of a relatively small group velocity, when particles aren't decelerated to it, about half of the particles leaves pulse decelerated (see Fig.4b). The efficiency in this case ~5 %.

4. Conclusion

The concept of the effective reflective amplification is described. The condition of all particles reflection are obtained. It was shown, that in the suggested schemes the particles energy extraction is significant. The opportunity of the amplification by several times was verified by results of numerical simulation in the case of non-resonant radiation. In our further work, we are planning to check and verify the same for the resonant amplification.

Acknowledgement

This work is supported by Russian Science Foundation (Grant no. 21-72-30027).

5. References

[1] Nanni E.A., et al., *Nature Communications,* **6,** 8486, 2015; doi: 10.1038/ncomms9486
[2] Tajima T., Dawson J.M. *Phys. Rev. Lett.*, **43,** 267, 1979; doi: 10.1103/PhysRevLett.43.267
[3] Kroll N.M., et al., *IEEE J. Quantum Electron.,* **17,** 1436, 1981; doi: 10.1109/JQE.1981.1071285
[4] Bandurkin I.V., Oparina Y.S., Savilov A.V. *Appl. Phys. Lett.*, **110**, 263508, 2017; doi: 10.1063/1.4990972
[5] Oparina Y.S., Savilov A.V., *Physics of Plasmas,* **28**, 093302, 2021; doi: 10.1063/5.0058758