

Thermal losses in the dielectric case of the absorbing load of liquid calorimeter in measuring the energy of high-power microwave pulses

A.I. Klimov*, V.Yu. Konev

Institute of High Current Electronics SB RAS, Tomsk, Russia

*klimov.1955@inbox.ru

Abstract. The problem of underestimation the result of measuring the energy of a high-power microwave pulse by a calorimeter with a disk-shaped absorbing load with an ethanol-based working fluid due to heat transfer to the input window of the load during the measurement is investigated. Estimates are made using the analytical half-space cooling model as well as computer simulation. It is shown that the underestimation of the measured microwave energy can be no more than one percent, which is much lower than the measurement error.

Keywords: liquid calorimeter, absorbing load, dielectric case, thermal losses.

1. Analytical estimations

Liquid calorimeters are successfully used to measure the energy of high-power microwave pulses [1]. The energy of the microwave pulse is determined by measuring the increase in the volume of the working fluid based on ethyl alcohol, filling the volume of the absorbing load. An advantageous feature of these devices is their calibration, which is performed by applying an electrical pulse to an active load (calibration heater) located in the volume of the working fluid. An analysis of the thermal processes associated with the calibration of the calorimeter was carried out in [2]. The process of absorption of microwave pulse energy in the working fluid may be accompanied by the leakage of part of the heat into the dielectric input window of the absorbing load during the measurement procedure and, thus, underestimation of the measured energy. This work is devoted to the analysis of this undesirable effect.

In calorimeters [1], to reduce the reflection of microwave radiation, the input window has a corrugated shape. However, to analyze the effect, it suffices to consider a flat geometry (Fig.1), since the most heat loss occurs when the energy release is concentrated near the dielectric.

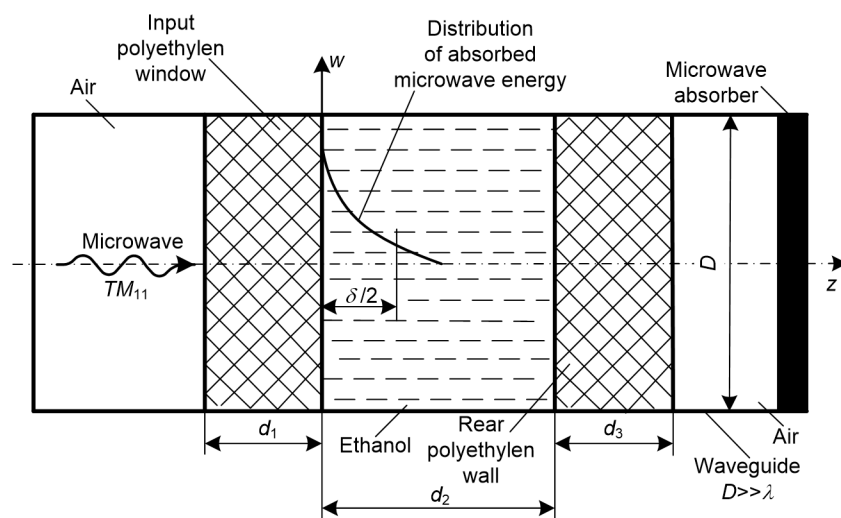


Fig.1. Plane geometry of the calorimeter model based on a circular waveguide with a diameter much more than the wavelength, $D \gg \lambda$.

Sufficiently rigorous analytical consideration of the heat transfer process is a difficult task. However, an analysis based on the use of the results of solving the well-known problem [3] on the cooling of a half-space (Fig.2) filled with a homogeneous medium with a temperature jump at the

initial moment of time can bring some clarity. Since the thermodynamic characteristics of polyethylene and ethyl alcohol do not differ very much, the use of the solution of this problem in the first approximation can be considered justified.

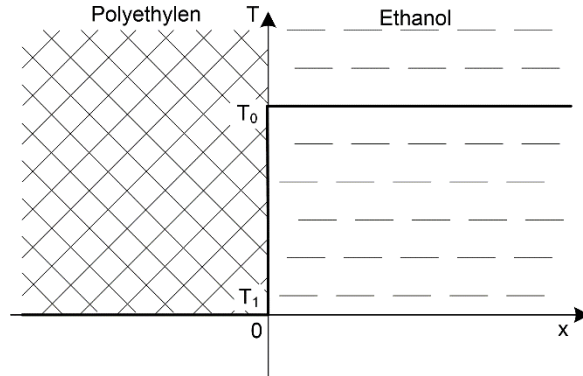


Fig.2. On the problem of cooling a half-space.

At the initial moment of time, $t = 0$, the temperature of the medium $T = T_0$ everywhere at $x > 0$. The temperature T_1 of the surface $x = 0$ is always constant, $T_1 < T_0$.

It is necessary to find the temperature distribution of the medium $T(x, t)$ in the half-space $x > 0$ for time points $t \geq 0$. The medium to the right of the $x = 0$ plane models ethyl alcohol in an absorbing load and has the physical characteristics corresponding to it. And the $x = 0$ plane itself corresponds to the interface between the alcohol and the dielectric of the input window.

The temperature distribution $T(x, t)$ is described by the heat conduction equation [3]:

$$\frac{\partial T}{\partial t} = \chi \frac{\partial^2 T}{\partial x^2}, \quad \chi = \frac{\kappa}{\rho c_v},$$

where χ is the thermal diffusivity, κ is the thermal conductivity, ρ and c_v are the density of alcohol and the specific heat of alcohol at constant volume, respectively.

The solution of the problem for $t \geq 0$ has the form:

$$T(x, t) = 2 \frac{T_0 - T_1}{\sqrt{\pi}} \int_0^{\frac{x}{2\sqrt{\chi t}}} e^{-\xi^2} d\xi + T_1, \quad \frac{\partial T(x, t)}{\partial x} = \frac{T_0 - T_1}{\sqrt{\pi \chi t}} e^{-\frac{x^2}{4\chi t}}, \quad \frac{\partial T(0, t)}{\partial x} = \frac{T_0 - T_1}{\sqrt{\pi \chi t}}.$$

Therefore, it is possible to estimate the dependences of the power flux density $p_0(t)$ and the energy flux density $\Delta w_0(t)$ on time through the liquid-dielectric interface, $x = 0$:

$$p_0(t) = \kappa \frac{\partial T(0, t)}{\partial x} = \kappa \frac{T_0 - T_1}{\sqrt{\pi \chi t}}, \quad \Delta w_0(t) = \int_0^t p_0(t') dt' = 2\kappa(T_0 - T_1) \sqrt{\frac{t}{\pi \chi}}. \quad (1)$$

In the estimates below, the following values of the characteristics of ethyl alcohol [4–6] for a temperature of 20 °C were used:

$$\rho = 789 \text{ kg/m}^3, \quad c_v = 2130 \text{ J/(kg} \cdot \text{K)}, \quad \kappa = 0.169 \text{ W/(m} \cdot \text{K)}, \quad c = 10^{-7} \text{ m}^2/\text{s}, \quad \text{and } c_p = 2510 \text{ J/(kg} \cdot \text{K)}$$

is specific heat capacity of alcohol at constant pressure.

The temperature increment $T_0 - T_1$, required to calculate the dependence $\Delta w_0(t)$, was estimated for two frequencies $f = 3 \text{ GHz}$ and $f = 10 \text{ GHz}$ on the axis of symmetry of a circular waveguide for a wave TM_{11} incident from air onto a layered structure (Fig.1). We considered a microwave pulse with a power amplitude $P = 580 \text{ MW}$, duration $\tau = 100 \text{ ns}$, and an energy of 58 J.

For frequency $f = 3$ GHz, $d_1 = 17.5$ mm, $d_2 = 45$ mm, $d_3 = 25$ mm, $D = 60$ cm, $\lambda = 10$ cm ($D \gg \lambda$). For frequency $f = 10$ GHz, $d_1 = 5.8$ mm, $d_2 = 45$ mm, $d_3 = 25$ mm, $D = 20$ cm, $\lambda = 3$ cm ($D \gg \lambda$). Since the condition $D \gg \lambda$, is satisfied in both cases, the dispersion of the waveguide can be neglected.

In both cases, the polyethylene thickness d_1 was optimized in the numerical simulations described below in order to minimize the microwave reflection coefficient K_{ref} . The alcohol thickness d_2 was sufficient for almost complete absorption of the microwave pulse energy, so that the reflection from the rear polyethylene wall of thickness d_3 could be neglected. For the frequency $f = 3$ GHz, $K_{ref} = -9.44$ dB, and for the frequency $f = 10$ GHz, $K_{ref} = -12.8$ dB.

The expression for the electric field of a plane wave propagating in a medium with absorption has the following form [7]: $E(z, t) = E_0 \exp[j(\omega t - kz)]$ where E_0 is the wave electric field amplitude, $\omega = 2\pi f = 2\pi c/\lambda$ is the angular frequency, f is the ordinary frequency, c is the speed of light in vacuum, λ is the wavelength. Since alcohol is a lossy medium, the wave number is complex: $k = k' + jk''$, where the values k' and k'' are

$$k', k'' = \pm \left(\frac{2\pi f}{c} \right) \sqrt{0.5 \left[\sqrt{\varepsilon'^2(f) + \varepsilon''^2(f)} \pm \varepsilon'(f) \right]}.$$

Here $\varepsilon'(f)$ and $\varepsilon''(f)$ are real and imaginary parts of the permittivity of alcohol, respectively. Thus, the electric field of the wave in alcohol decays in accordance with the relation: $E(z, t) = E_0 \exp[j(\omega t - k'z)] \exp(-k''z)$. Value $\delta = 1/|k''|$ is the spatial scale of the field damping. Since the power flux density of the wave is $\sim |E(z, t)|^2$, the spatial scale of the decrease in the absorbed power density and the absorbed energy density is equal to $\delta/2 = 1/(2|k''|)$, i.e., two times smaller than the spatial scale of the field attenuation. Therefore, $p(z) = p_0 \exp(-2z/\delta)$, where p_0 is the power flux density at the dielectric-alcohol interface.

For calculations we used the values for $\varepsilon'(f)$ and $\varepsilon''(f)$, obtained by approximation by Debye's formula, respectively, the real and imaginary components of the dielectric permittivity of ethyl alcohol with a concentration of 95 %, based on data from [8].

For $f = 3$ GHz, $\varepsilon'(f) = 10.3$, $\varepsilon''(f) = 8.8$, $\delta \approx 1.25$ cm. For $f = 10$ GHz, $\varepsilon'(f) = 5.7$, $\varepsilon''(f) = 3.4$, $\delta \approx 0.698$ cm. For polyethylene $\varepsilon'(f) = 2.26$, $\varepsilon''(f)/\varepsilon'(f) = 0.0004-0.0005$ [9], and the absorption of microwave energy in the input window can be neglected.

The power ΔP absorbed in a thin layer $\Delta z \ll \delta/2$ of the working fluid within a small circle of area S near the waveguide axis (Fig.1) is

$$\Delta P = S[p(z) - p(z + \Delta z)] = p_0 S \left(e^{\left(-\frac{2z}{\delta}\right)} - e^{\left(-\frac{2}{\delta}z\right)} e^{\left(-\frac{2}{\delta}\Delta z\right)} \right) \approx p_0 S e^{\left(-\frac{2z}{\delta}\right)} 2\Delta z / \delta.$$

Near the window, $z = 0$, and $\Delta P = 2p_0 S \Delta z / \delta = 2p_0 V / \delta$, where $V = S \Delta z$ is the volume of the layer. The energy absorbed in the layer is

$$\Delta P \tau = 2p_0 \tau V / \delta = mc_p (T_0 - T_1) = \rho V c_p (T_0 - T_1), (T_0 - T_1) = (2p_0 \tau) / (\delta \rho c_p), \quad (2)$$

where $m = \rho V$ is the mass of the alcohol layer.

The power flux density p_0 was estimated using a formula that relates the microwave power P^{tr} and the electric field strength E of the wave TM_{11} on the axis of the circular waveguide [10]. In the case of $D \gg \lambda$, $E_{kV/cm}^2 = 6320 P_{MW}^{tr} / (\pi D_{cm}^2)$. In this case the wave front near the waveguide axis is close to plane, so we used the formula linking field strength and power flux density for a plane wave: $E_{kV/cm}^2 = 729 p_{0,MW/cm}^2$, where $p_{0,MW/cm} = 8.67 P_{MW}^{tr} / (\pi D_{cm}^2)$.

For frequency $f = 3$ GHz, and $D = 60$ cm, $K_{ref} = -9.44$ dB, total power absorbed in alcohol is $P^{tr} = 514$ MW, total energy absorbed over the entire pulse duration is $W = p\tau = 51.4$ J, power density

at the axis is $p_0 = 0.394 \text{ MW / cm}^2$, total density of absorbed energy is $w_0 = p_0\tau = 3.94 \cdot 10^{-2} \text{ J/cm}^2$. For frequency $f = 10 \text{ GHz}$, and $D = 20 \text{ cm}$, $K_{ref} = -12.8 \text{ dB}$, total power absorbed in alcohol is $P^{tr} = 550 \text{ MW}$, total absorbed energy is $W = 55 \text{ J}$, and the values p_0 and w_0 are as follows $p_0 = 3.79 \text{ MW / cm}^2$, $w_0 = 0.379 \text{ J/cm}^2$.

Using ratio (2), one can estimate the temperature increment at the boundary between the working fluid and the dielectric. For frequency $f = 3 \text{ GHz}$, $(T_0 - T_1) = (2p_0\tau)/(\delta\rho c_p) \approx 0.032 \text{ K}$. For frequency $f = 10 \text{ GHz}$, $(T_0 - T_1) = (2p_0\tau)/(\delta\rho c_p) \approx 0.55 \text{ K}$.

Using the substitution of these temperature differences in ratio (1), the values of the relative heat loss $\Delta w_0/w_0$ from the working fluid to the dielectric for various time intervals were calculated. These values are given below in Table 1.

2. Computer simulation

Computer simulation of the thermal processes was performed using the CST Microwave Studio software. The simulation geometry (Fig.1) and the initial data of the problem, including the parameters of the microwave pulse, are given above. As in the analytical estimates, at frequencies of 3 and 10 GHz, the TM_{11} wave incident on the layered structure was specified. The power fluxes were calculated through the entire cross section of the waveguide. For both frequencies, the reflection coefficients of the microwave power from the structure were calculated. The calculation of the structure of electric and magnetic fields arising during the microwave pulse as well as the distribution of microwave energy absorbed in the working fluid near the input window was successively performed. Then, using the Thermal module, computer simulation of thermal processes was performed. This simulation included the calculation of the distribution of absorbed energy, the heat flux into the input window, convection currents, as well as changes in the heat flux and temperature of the working fluid with time.

3. Analysis of the results

The calculated maximum initial temperature increase near the boundary between the dielectric and the working fluid are in satisfactory agreement with the results of analytical estimates. Fig.3 presents the results of computer calculations of power flows from the working fluid into the input window for various times.

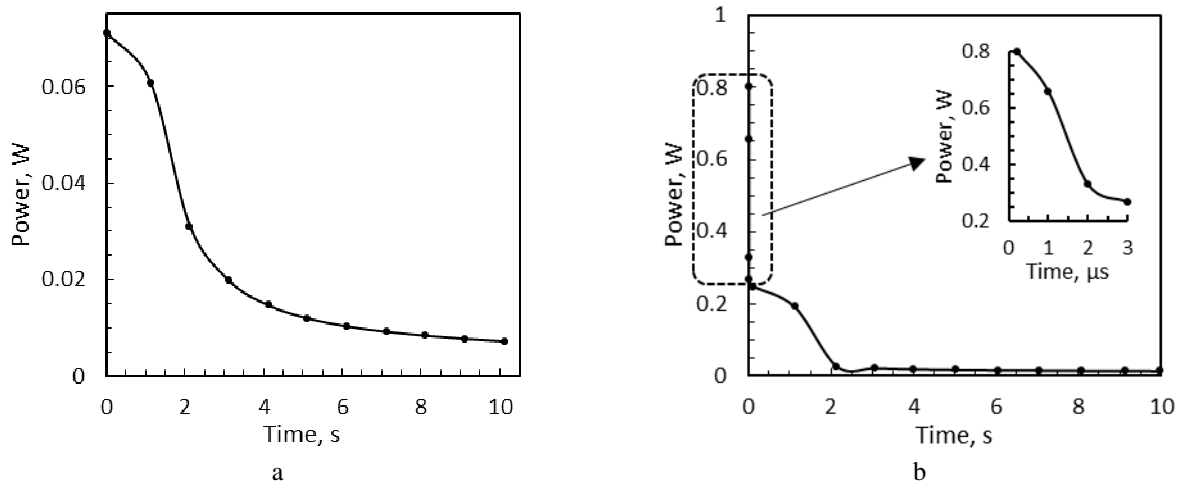


Fig.3. Power going into the input window for 3 GHz (a) and 10 GHz (b).

It can be seen that both dependences are noticeably different from (1) and have a region with a pronounced sharp decline. The characteristic time for the decrease in the power flux is $\approx 3 \text{ s}$ for a frequency of 3 GHz and $\approx 2.5 \mu\text{s}$ for a frequency of 10 GHz. A sharp decrease in the power going into

the dielectric can be explained by a decrease in the temperature gradient over time, as well as the emergence and development of convection, which removes heat from the dielectric into the volume of alcohol (Fig.4). These processes are more pronounced in the case of a frequency of 10 GHz, which may be due to the fact that the temperature difference $T_0 - T_1$ of the working fluid in this case is an order of magnitude larger than in the case of 3 GHz. The data presented in Fig.3 allowed us to calculate the relative heat loss $\Delta W/W$ into the window for various periods of time. The results of these calculations are shown in Table 1. From (1) and the data of Table 1, it can be seen that the analytical estimates of the heat loss are more than an order of magnitude higher than the results of computer simulation. The estimates and the computer simulations show that thermal losses for a frequency of 10 GHz exceed those for a frequency of 3 GHz, which can be explained by a higher difference $T_0 - T_1$ at the boundary between the working fluid and the dielectric in the case of a frequency of 10 GHz. The relative heat leakage into the window, obtained as a result of the computer simulation, is significantly less than that obtained in the analytical estimates and better corresponds to the results usually observed in the measurements [1]. The computer simulation more accurately takes into account the processes of the heat transfer and, therefore, gives more correct results. The half-space cooling model may be of limited use for a rough estimate, and only for a time interval of a few seconds. It should be taken into account that in this case, it gives more than an order of magnitude overestimated value of heat losses.

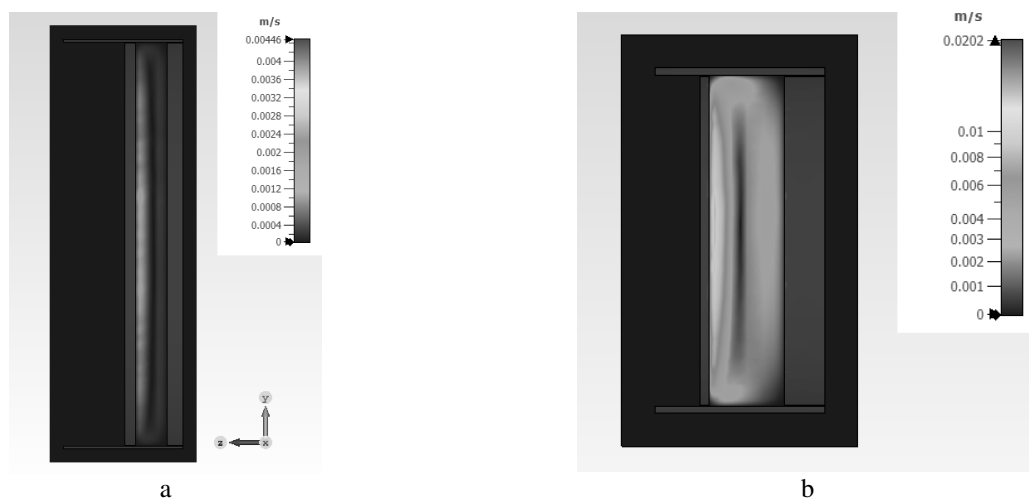


Fig.4. Patterns of convection in the working fluid in the case of a frequency of 3 GHz for a time instant of 6 s (a) and 10 GHz for time 2 s (b).

Table 1. Results of analytical estimations and computer simulations

Analytical estimations				Computer simulations			
$f = 3 \text{ GHz}$		$f = 10 \text{ GHz}$		$f = 3 \text{ GHz}$		$f = 10 \text{ GHz}$	
$t, \text{ s}$	$\Delta w_0/w_0, \%$	$t, \text{ s}$	$\Delta w_0/w_0, \%$	$t, \text{ s}$	$\Delta W/W, \%$	$t, \text{ s}$	$\Delta W/W, \%$
1.1	5.1	1.1	9.1	1.1	0.14	1.1	0.4
3.1	8.6	3.1	15.4	3.1	0.28	3.1	0.65
5.1	11.0	5.1	19.1	5.1	0.34	5.0	0.72
7.1	13.0	7.1	23.2	7.1	0.38	7.1	0.79
10.1	15.5	10.1	27.7	10.1	0.43	10.0	0.87

4. Conclusion

The performed studies of heat transfer into the input window of a flat disk-shaped absorbing load of a calorimeter with an ethanol-based working fluid show that the relative underestimation of the measured energy of a high-power microwave pulse can be fractions of a percent. This value is much

less than the relative measurement error, which is usually $\pm(10-20)\%$ [1]. The results obtained can be approximately applicable to absorbing loads with a corrugated input window, since the absorption of microwave energy occurs mainly in a rather narrow layer near the dielectric, and it can be assumed that the nature of the heat transfer processes does not differ dramatically from that in the case of a flat geometry of the absorbing load.

5. References

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