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# Terahertz free electron laser with an electrodynamic system based on the excitation of Talbot-type supermode

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**Abstract.** In this work we describe the concept of an electron maser based on the excitation of the single supermode formed by a fixed set of transverse eigenmodes is proposed. The high-Q supermode is proposed as an operating mode. This mode is formed inside a simple cavity by a set of its eigenmodes. The supermode is high-Q due to the Talbot effect, namely, periodic reproduction of the transverse structure of a multimode wave field in an oversized waveguide. Two mirrors at the input and output of the waveguide provide the mode localization inside the cavity.
**Keywords:** free electron maser/laser, terahertz radiation.

## 1. Introduction

Sources operating in the terahertz (THz) and sub-THz frequency ranges with a high power of the output radiation are required in various actual applications. For instance, they are used for heating and current drive or diagnostic systems of fusion installations of the next generation, such as DEMO, and for many actively developing areas, such as high-gradient THz acceleration, sub-THz wave undulators for short-wavelength free-electron lasers, and various plasma physics applications.

There are several problems to generate powerful coherent THz radiation in masers based on relativistic electron beams (free electron masers, FEMs). At first, such devices have to operate on excitation of the one far-from-cutoff transverse mode of the operating waveguide, as a rule, this is the lowest possible transverse mode (this is to solve the problem of the mode selection). Difficulties appear in the THz frequency range. Evidently, the operating waveguide in this case should be oversized [1, 2]; this means that the characteristic transverse size should be much greater than that of the operating wavelength. This is necessary for several reasons, namely, transportation of the relativistic high-current beam, the problem of breakdown of the field of high-power radiation inside the cavity, ohmic heating of the cavity walls, etc. The second problem is the difficulty in providing selective single-mode feedback in an oversized system [3].

We propose the concept of selective excitation of a THz operating wave in a high-power relativistic electron maser with an oversized microwave system fed by a high-current relativistic electron beam. Our idea is to give up working in a fixed transverse mode. Instead, we propose to work in a supermode, which is formed by a fixed spectrum of several transverse modes of an oversized waveguide, Fig.1.

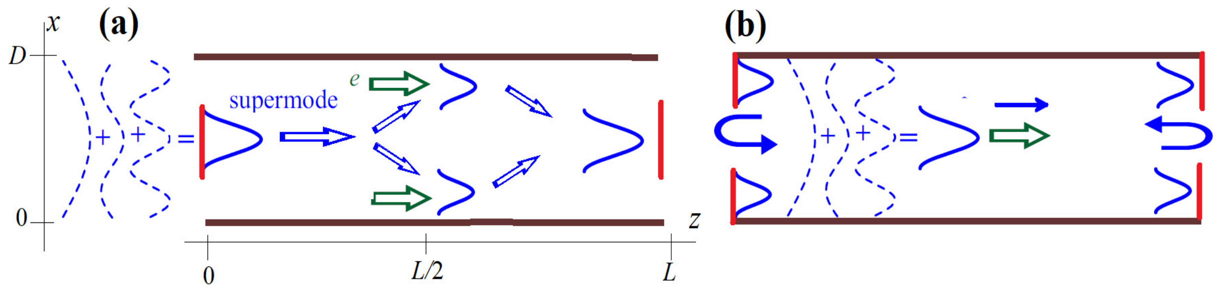


Fig.1. Two possible schemes of excitation of a supermode formed by a set of several transverse modes. The (a) is obtained, (b) by its shifting by half of the space period.

## 2. The Talbot-type supermode formation

Consider the wave packet possessing a fixed frequency and propagating in a waveguide can be represented as a sum of fields of partial transverse eigenmodes of this waveguide (Fig.1):

$$\boldsymbol{E}_{\Sigma} = \mathrm{Re}\sum_{n} C_n \boldsymbol{E}_n(\boldsymbol{r}_{\perp})\exp(i\omega t - ih_n z), \tag{1}$$

here, $n$ is the transverse mode number, $\boldsymbol{E}_n(\boldsymbol{r}_\perp)$ describes transverse structures of the $n^{\text{th}}$ mode, $h_n = (k^2 - k_{\perp,n}^2)^{1/2}$ are the axial wavenumbers, $k = \omega/c = 2\pi/\lambda$, and $k_n$ are the transverse wavenumbers. We consider the situation of a highly oversized system, $D/\lambda \gg 1$. In this situation, a great numbers of partial transverse modes can be involved in the formation of the wave packet, and most of the modes are very far from the cutoff, $k_{\perp,n} \ll k$. Therefore, their axial wavenumbers can be approximated as follows:

$$h_n \approx k - \frac{k_{\perp,n}{}^2}{2k} = k - \frac{\pi\lambda n^2}{4D^2}. \tag{2}$$

The phase incursion of the mode at its trip along the waveguide of the length $L$ is equal to

$$\varphi_n = h_n L \approx kL - \frac{\pi n^2}{4} \times \frac{\lambda L}{D^2}. \tag{3}$$

Consider transverse distribution of the wave field, $E(x) = E(D - x)$, is formed by modes having odd indices, $n = 2m - 1$, here $m$ is an integer positive number. In this case, field is a “spot” concentrated to the center at the input and the output of the cavity (see Fig.1a). For these modes,

$$\varphi_n = kL - \frac{\pi}{4} \times \frac{\lambda L}{D^2} - \pi m(m-1) \times \frac{\lambda L}{D^2},$$

obviously, that $m(m - 1)$ is an even number, so transverse distribution of the wave field is reproduced in the waveguide (Fig.1), $\boldsymbol{E}_\Sigma(x,z + L) = \boldsymbol{E}_\Sigma(x,z)$, with a period described by the following formula:

$$L = \frac{D^2}{\lambda}. \tag{4}$$

The phenomenon of repetition of the transverse wave structure is well known as the Talbot effect [4]. In paper [5] the use of this effect is proposed as a way to create an oversized microwave system of an electron maser that provides a high Q-factor for a supermode formed by several transverse modes. If relatively narrow mirrors at the input and the output in the centers of the corresponding transverse cross-sections will be include in the system, the field of a high-Q supermode, $\boldsymbol{E}_\Sigma(x)$, is concentrated only in the region of the mirror. If the length of the cavity satisfies formula (4) and the Talbot effect is executed ideally, then this transverse profile of the total wave field is reproduced exactly at the output transverse cross-section ($z = L$). In this case, this wave field is reflected completely by the output mirror. The counter-propagation of the reflected wave back to the input mirror is completely analogous to the direct propagation of the supermode. Therefore, the input mirror completely reflects the counter-propagating wave into the forward wave and closes the feedback circuit. Thus, a simple cavity provides a high Q-factor for any supermode, which electric field $\boldsymbol{E}_\Sigma(x)$ in the input/output transverse cross-sections of the cavity is concentrated in the regions of the mirrors, and, therefore, almost totally reflected back to the cavity. This high-Q supermode is a specific set of transverse waveguide modes, (Fig.1a). Another scheme of a Talbot-type cavity can be formed by “shifting” the scheme shown in the Fig.1a along the $z$-axis by length

*L*/2. In this case, the supermode field is two “spots” concentrated close to the walls at the input and the output of the cavity (Fig.1b). One more attribute of the Talbot effect is multiplication of the wave beam. This means that, in the middle of the cavity shown in (Fig.1a), the transverse distribution of the supermode field represents two wave beams concentrated close to the waveguide walls. Therefore, such a supermode can be excited effectively by two electron beams injected in the regions close to the walls. In the case shown in the Fig.1b, a supermode is excited effectively by a single electron beam in the center. Therefore, the problem of separation of the electron beam and the input and output mirrors is easily solved. The analogue for cylindrical geometry of the scheme is a waveguide of the circular cross-section. There are several distinctive features caused by non-equidistance of the spectrum of transverse modes [6].

**3. Simulations of THz-Frequency-Range High-Current FEM Oscillators**

Accurately, the Talbot effect occurs only in systems with an equidistant spectrum of transverse eigenmodes. In a waveguide with a circular cross section, the mode spectrum is quasi-equidistant. Nevertheless, the formation of a high-Q supermode is possible. In a waveguide with a circular cross section, the mode spectrum is quasi-equidistant (the spectrum of eigen modes with the azimuthal index “1” and various radial indexes); this is true only for sufficiently high transverse modes. Accordingly, the Talbot effect occurs only approximately. Nevertheless, the formation of a high-quality supermodel is possible.

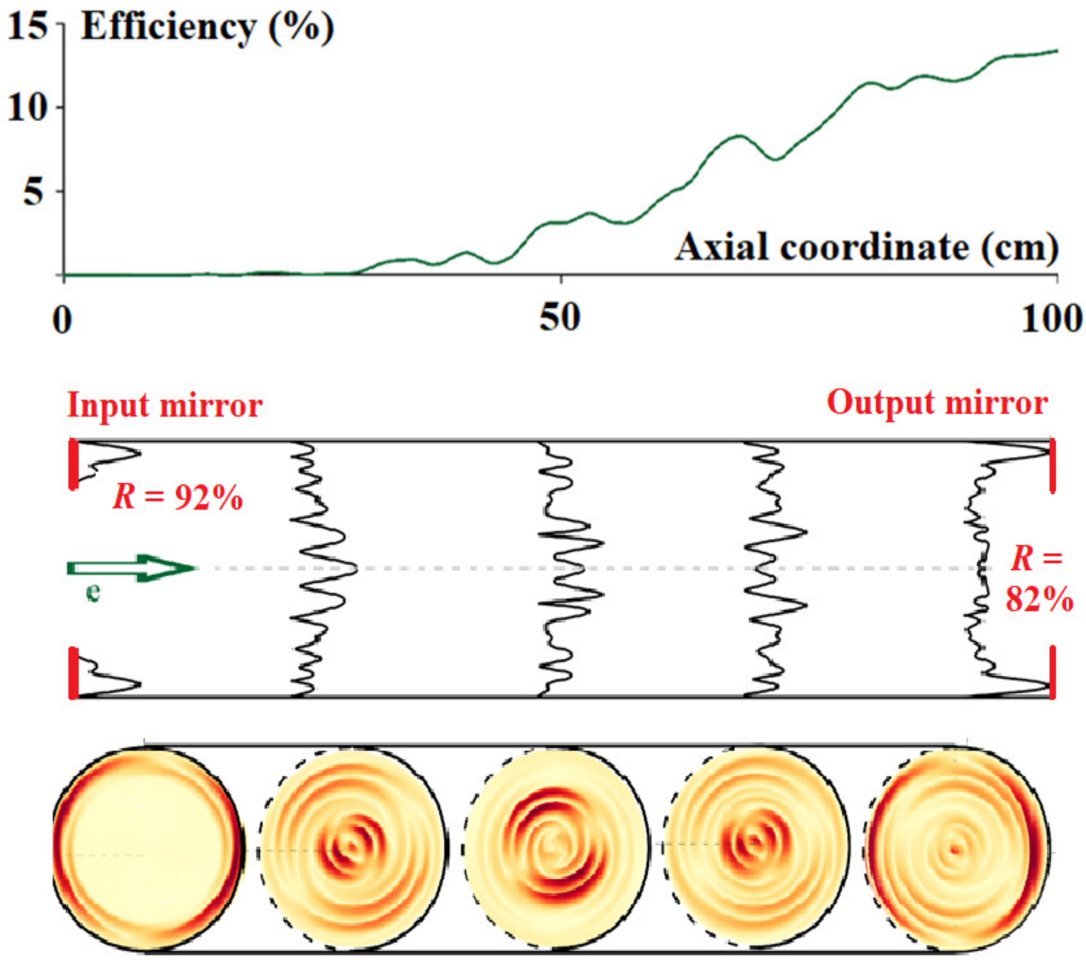


Fig.2. 0.3 THz FEM. Electron efficiency versus the axial coordinate inside the cavity in the steady-state regime, and calculated wave structure of the supermode excited inside the cavity.

We study the possibility for the realization of a THz frequency-range FEM oscillator based on the LIU (the Budker Institute for Nuclear Physics [7]). The goal of this work is to provide the formation of a high-current (up to 2 kA) electron beam with electron energy up to 20 MeV and an electron pulse duration of 200–300 ns. The expected thickness of the electron beam is several mm. Therefore, the diameter of the operating waveguide should be at least $D = 1$ cm.

For the bunch with 2 kA, 5 MeV, 0.3 THz FEM based on the excitation of a Talbot-type supermode in a long oversized cavity $D = 36$ mm with the circular cross-section. The factor of transverse oversizing in this system, $D/\lambda$ is about of 40. Simulations of the self-excitation of the Talbot-type supermode in such a system were carried out on the basis of the self-consistent approach described in [6]. In the FEM with helical undulator and axis-encircling electron beam, electrons interact only with modes $TE_{1,n}$ and $TM_{1,p}$. These partial modes form the "hot" structure of the supermode. According to simulations, the FEM is excited quite fast (during ~20 trips of the wave through the cavity; this corresponds to a time ~100 ns). In the saturated regime of the stationary generation of this FEM, the calculated electron efficiency (~15%) corresponds to the output wave power exceeding 1 GW (Fig.2).

Fig.3 illustrates simulations of the 2 kA, 7 MeV, 2 THz FEM based on the excitation of a Talbot-type supermode in an oversized cavity ($D = 1$ cm). In this case, the factor of transverse oversizing, $D/\lambda$, is over 60, and the length of this system is $L = 65$ cm. The electron efficiency in the steady-state regime of excitation of the TE-like supermode amounts to approximately 8%. This corresponds to an output power close to 1 GW at the frequency 2 THz. The typical duration of the transition process is about 30 round trips of the wave through the cavity (about 100 ns).

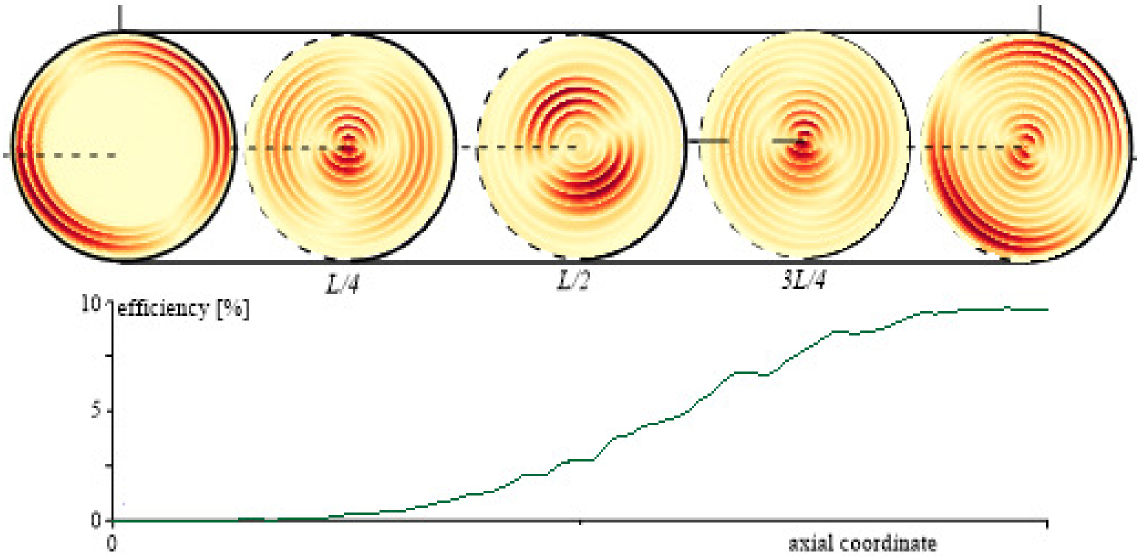


Fig.3. 0.3 THz FEM. Electron efficiency versus the axial coordinate inside the cavity in the steady-state regime, and calculated wave structure of the supermode excited inside the cavity.

**4. Simulations of the "cold" measurements experiment**

In order to prepare electrodynamic system for the FEL, we have to carry out experiment for verifying of the mode formation. The scheme for this experiment is shown in the Fig.4a. There is the cylindrical waveguide, the length and diameter of which is chosen to ensure formation of the high-Q Talbot type supermode with the spectral composition shown in the Fig.4b. There is a small hole for the detection of the field maximum formation in the mirror at the output cross section. The mode $TE_{18}$ prevails in the input signal for the better mode transformation into the mode $TM_{17}$. The hole doesn't break the supermode structure, so the peak of the detected power is observed, Fig.4c, at the predicted frequency, which accords to the maximum of the reflection coefficient, Fig.4d.

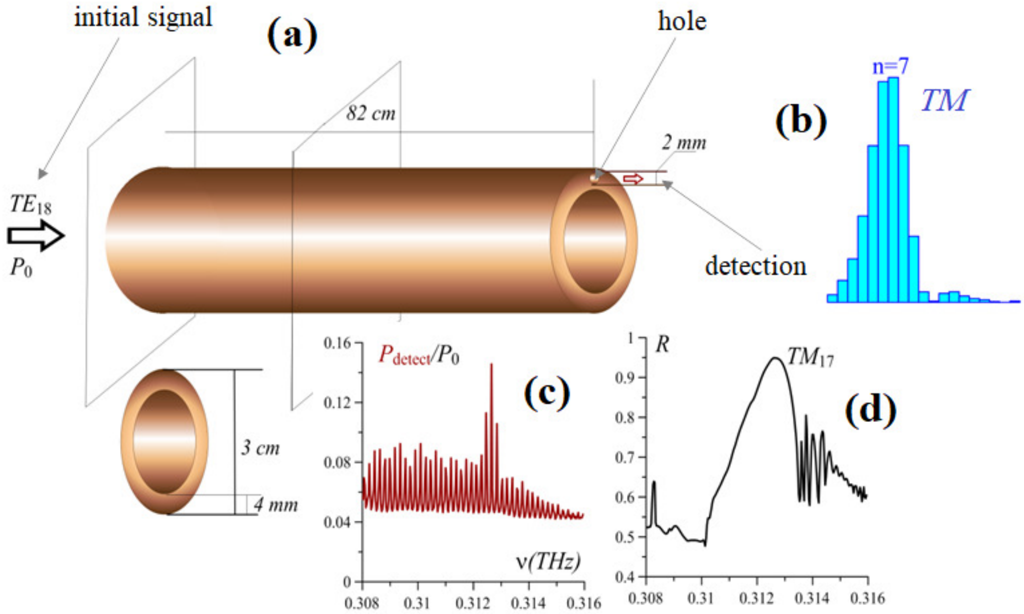


Fig. 4. Scheme of the "cold" measurements experiment.

## 5. Conclusions

The theory of free electron laser with an electrodynamic system based on the excitation of Talbot-type supermode is developed. The obtained results is attractive from the viewpoint of the practical realization.


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