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Conduction current in low-density plasma opening switches

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Abstract. The paper analyzes two concepts of conduction in a low-density plasma opening switch: based on the formation of a bipolar space charge layer subject to unlimited cathode emissivity (bipolar model) and of a unipolar ion layer due to electron motion off the cathode under an electric field induced by magnetic field penetration into the plasma (unipolar model).
A comparison of unipolar-model expressions with widely known experimental data shows that the conduction current behaves strictly as $I_c \propto n^{1/2}$ at any plasma bridge length in the range of plasma densities from ~10^{11} cm^{-3} to ~10^{14} cm^{-3} and current rise rates from ~0.3 kA/ns to ~4 kA/ns. Holding the conduction current constant at any bridge length l requires that nl^2 be constant. As the magnetic field rise rate is varied, the conduction current follows $I_c \propto \dot{B}^{1/2}$. These results differ radically from what is predicted by the bipolar model. Also presented in the paper are arguments of why the axial current channel width varies nonmonotonically during a pulse and estimates of the electron temperature at different plasma densities and field rise rates.
Keywords: plasma opening switch, conduction current, self-magnetic insulation.

1. Introduction

On the assumption of a bipolar space charge layer near the cathode of a plasma erosion opening switch [1], the conduction current is given by $I_c = j_{sat}S$, where j_{sat} is the saturation current density and $S = 2\pi r_c l$ is the cathode plasma area subject to unlimited emissivity with r_c for the cathode radius and l for the axial plasma length (Fig.1). In essence, such a bipolar model suggests that the emission current behaves as $j_{sat} = (m_i/Zm)^{1/2}j_i$, where m_i, m are the ion and the electron mass and $j_i = Zen_iv_{inj}$ is the ion current density due to a directed plasma flow velocity v_{inj} with Ze for the ion charge and with $n = Zn_i$.

Putting the conduction current equal to the bipolar saturation current is incorrect for several reasons. First, it gives a paradoxical magnetic field velocity u. Actually, if $I_c = j_{sat}S$, $I_c = \dot{I}t_c$, $t_c = l/u$, we have $u = \dot{B}c/4\pi j_{sat}$ with c for the velocity of light such that $u \propto v_{inj}^{-1}$, which has neither experimental [2] nor simulation evidence [3]. Second, the formula for the saturation current is independent of the magnetic field rise rate, which is inconsistent with experiments. Third, for j_e/j_i to be constant, several conditions should hold. In particular, the electric field at both boundaries of the collisionless space charge double layer should be zeroed. In addition, the ratio of the currents depends on the rate of ion supply in the bipolar layer. Obviously, these conditions in plasma opening switches are not valid, and the ratio j_e/j_i during the formation of the layer can greatly differ from its Langmuir value [4].

The above arguments suggest searching for an alternative concept to adequately explain the fact of faster-than-diffusion magnetic field penetration into a fully ionized plasma. Presented below is a brief description of such a concept [5] and a detailed comparison of its predictions with experimental data.

For simplicity and clear presentation, all analytical relations are given in CGS units because the electric and magnetic fields in the CGS system have the same dimension. In numerical examples, the current is given in amperes which are easily convertible to CGS.

2. Concept of unipolar layer formation

The proposed concept (unipolar model) admits the formation of a unipolar ion layer as electrons are driven off the cathode by an induced electric field $E_{ind}(t)$ due to magnetic field penetration into the plasma (Fig.1). For low-density plasmas, the main points of the concept are the following.

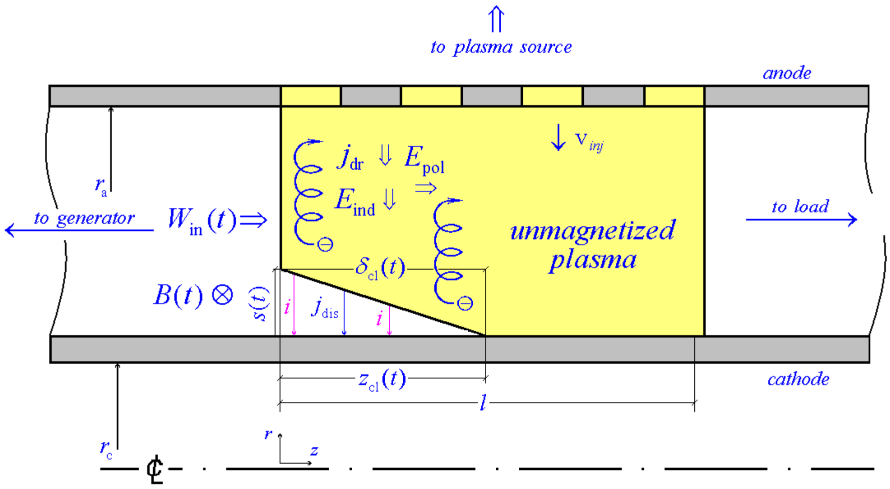

Fig.1. Switch schematic with electron paths (out of scale).

In a collisionless plasma, the electron magnetization time at an increasing magnetic field measures $\tau_{cl} = (3mc/e\dot{B})^{1/2}$. Further, the current transport across the magnetic field is provided by radial electron drift in crossed magnetic and polarization electric fields. The drift velocity is given by $v_{dr} = \alpha u$, where $\alpha = 3/2$ and $\alpha = 4/\pi$ for collisionless and collisional plasmas, respectively. The polarization field E_{pol} results from charge separation as electrons move in the current channel. It balances the Ampere force on the electrons. As a result, the velocity of field penetration into the plasma is $u_{cl} = (\dot{B}c/6\pi en)^{1/2}$. The switch conduction current at this velocity is given by

$$I_c = (3\pi enc\dot{B}r_c^2 l^2/2)^{1/2}. \tag{1}$$

In a collisional plasma, the time of electron magnetization and transition to drift measures $\tau_d = mc\nu_{ei}/e\dot{B}$. In this case, unlike the collisionless one, τ_d depends on the electron collision frequency ν_{ei}. The velocity of field penetration is given by $u_d = (\dot{B}c/16en)^{1/2}$, and the switch conduction current by

$$I_c = (4enc\dot{B}r_c^2 l^2)^{1/2}. \tag{2}$$

Despite the dependence of τ_d on the electron collision frequency, the velocities u_{cl} and u_d differ slightly and only in their numerical factor. Thus, the conduction currents in the two cases are given by almost identical formulas (1), (2).

In the collisionless case, the electrical conductivity of the current channel due to electron magnetization is given by $\Sigma(t) = j_{dr}/E_{ind}(t) = \sigma_0/\omega_e(t)\tau_{cl}$ with

$$\sigma_0 = 3\omega_{pe}^2\tau_{cl}/8\pi,$$

where ω_{pe} is the electron plasma frequency. Relating the induced electrical field and codirectional current density, this expression for the channel conductivity describes its inversely proportional dependence on the electron magnetization parameter $\omega_e(t)\,\tau_{cl}$, where $\omega_e(t)$ is the electron cyclotron frequency. In the collisional case, the channel conductivity is given by $\Sigma(t) = \sigma_0/\omega_e(t)\tau_{ei}$, where $\sigma_0 = 4\sigma/\pi$ is close to the Spitzer conductivity σ.

At this conductivity, the energy delivered to the switch $W_{in}(t) = 6^{-1}B^2(r_c,t)z(t)r_c^2\ln(r_a/r_c)$ is spent equally for creating a magnetic field in the plasma volume and for ohmic heating [4]. The switch resistance $R_s = u\ln(r_a/r_c)/c^2$ is proportional to the velocity of magnetic field penetration u equal either to u_{cl} or to u_d.

For the magnetic viscosity $v_m(t) = c^2/4\pi\Sigma(t)$, the current channel width in the volume of collisionless plasma $\delta_{cl}(t) = [v_m(t)t]^{1/2} = u_{cl}t$ and collisional plasma $\delta_d(t)=u_dt$ increases in sync with the depth of field penetration along the cathode $z_{cl}(t) = u_{cl}t$ or $z_d = u_dt$.

The above relations for the current channel conductivity and width correspond to the experimental fact of faster-than-diffusion magnetic field penetration into the plasma of opening switches with a conduction time of ~50 ns [6]. First, the velocity of such penetration is almost constant during a linear current rise. Second, the magnetic field front penetrates the plasma at about the same and mainly radial current density. Third, the axial dimension of the current channel is much larger than the collisionless electron skin layer, such that its value can reach about half the initial bridge length.

3. Comparison with experiment

Let us refer to well-known pioneering experiments [7–9] which demonstrate the unique capabilities of plasma opening switches to interrupt the current and amplify the power at a load.

In Gamble I experiments, the current rise to ~200 kA in ~50 ns; the plasma bridge length was 3 cm at a cathode radius of 2.5 cm. Fig.2 shows the conduction current versus the delay time t_d between the current pulse and plasma injection in the Gamble I experiments [10, Fig.9] and its values along with those of the ion density and plasma injection velocity predicted by the bipolar model.

Fig.3 shows the conduction current versus the plasma density according to the Gamble I experiments and to unipolar-model expression (1). As can be seen, the conduction current behaves strictly as $I_c \propto n^{1/2}$. At the same time, the plasma density is ~5 times higher than the values in Fig.2. However, this difference is inessential because n_i and v_{inj} vary simultaneously in Fig.2. It is also seen that the estimate by formula (1) depends neither on the injection velocity v_{inj} nor on the Langmuir ratio $(m_i/Zm)^{1/2}$. These facts are of crucial importance. It should be noted that $I_c \propto n^{1/2}$ at v_{inj} measuring ~10 cm/µs to ~600 cm/µs was noted even in Omni II experiments [2].

To exclude any element of chance in the correlation of experimental and analytical data, let us refer to Gamble I and *proof-of-principle* (POP) experiments whose results at different delay times and bridge lengths (12 cm for POP) are reproduced in Fig.4 from Fig.8 presented elsewhere [11]. From analysis of these results [6, Fig.9] it follows that the conduction current does not depend on the bridge length l if the product nl is constant, which fits the bipolar model.

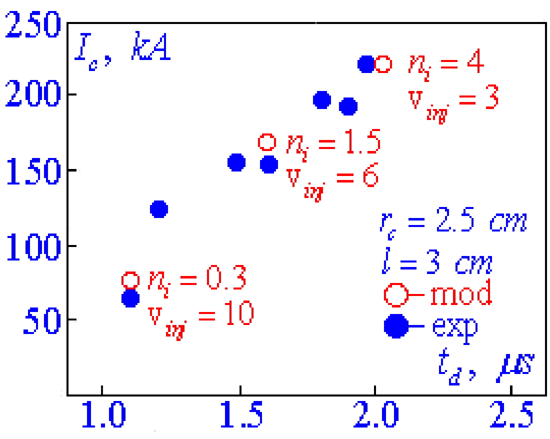

Fig.2. Conduction current I_c versus delay time t_d according to Gamble I experiments and bipolar model

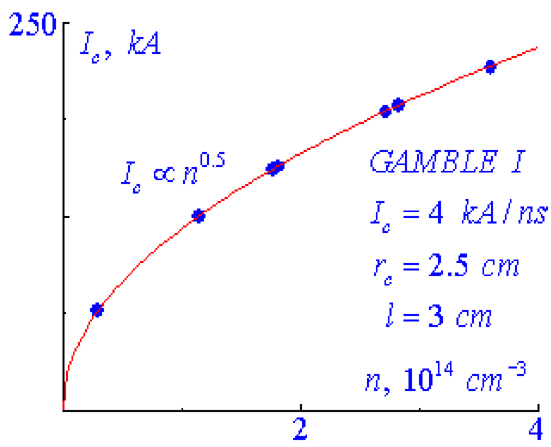

Fig.3. Conduction current versus plasma density according to formula (1).

(n_i in 10^{13} cm^{-3}, v_{inj} in cm/μs) [10].

However, applying formula (1) to the above Gamble I results identifies a radically different behavior of the conduction current. As can be seen from Fig.5a, the plasma density varies from ~$8\cdot10^{10}$ cm^{-3} to ~$4\cdot10^{13}$ cm^{-3} (three orders of magnitude). The conduction current at any bridge length behaves strictly as $I_c \propto n^{1/2}$, and at the same plasma density, $I_c \propto l$. For the conduction current to be constant at any bridge length, nl^2 should be constant. Thus, applying formula (1) to the experimental data of Fig.4 breaks them into a fan of $I_c \propto n^{1/2}$ with different l.

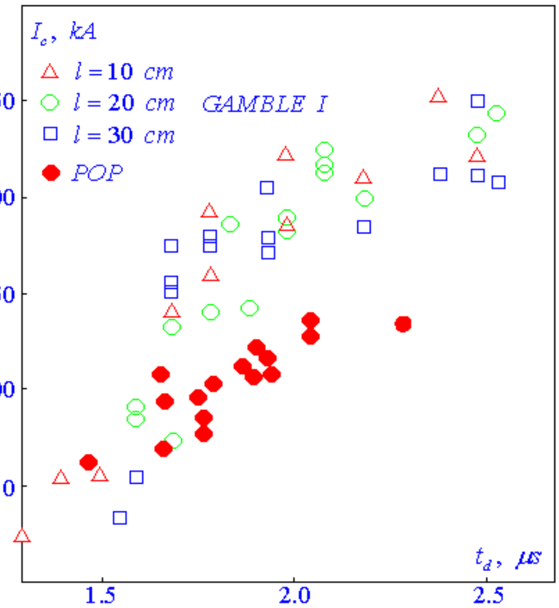

Fig.4. Conduction current from Gamble I and POP experiments [11].

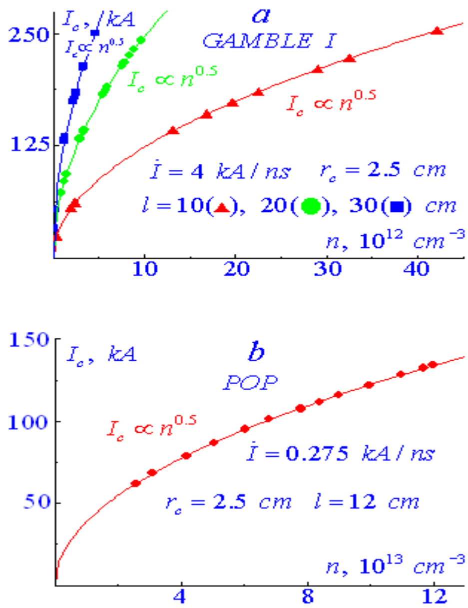

Fig.5. Conduction current versus plasma density according to formula (1).

If we look at the POP results (Fig.5b), we can see that the rate of current rise is about 15 times lower than its Gamble I values. Therefore, bringing the conduction time to ~500 ns requires a plasma density of about $1.2\cdot10^{14}$ cm^{-3}. This density is close to and yet lower than ~$1.4\cdot10^{14}$ cm^{-3} for the plasma to be collisional. Thus, formula (1) holds even in the case of POP, attesting that $I_c \propto n^{1/2}$ up to a conduction time of ~500 ns (Fig.5b). The same tendency ($I_c \propto n^{1/2}$.) was observed in Eyess experiments at $I_c \approx 500$ kA, $t_c \approx 500$ ns and in Falcon experiments at $I_c \approx 300$–700 kA, $t_c \approx 150$–350 ns [12]. Such a result was explained using a modified bipolar model [12] which identified the near-cathode space charge layer width with the electron Larmor radius and the magnetic insulation current with the conduction current.

4. Current channel width

Let us analyze the plasma dynamics in terms of plasma acceleration and aggregation.

The plasma can be taken collisionless subject to $\omega_{pe}^2 < 4\pi\sigma/\tau_{cl}$, which suggests that the collisionless electron skin layer surpasses the diffusion depth within τ_{cl}. If the plasma density satisfies $c/\omega_{pe} \leq (\beta Zm/m_i)^{1/3}(r_a-r_c)$, where $\beta = 1/9$ and $\beta = \pi/24$ for collisionless and collisional plasmas, respectively, the axial displacement of ions by the polarization field within the time of electron drift across the interelectrode gap $\tau_{dr} \approx (r_a-r_c)/v_{dr}$ is larger than the collisionless electron skin layer. Thus, the plasma motion becomes significant, the radial current flow and its axial displacement are superimposed, and the current transport in the switch is by illusion perceived as diagonal.

In addition, such axial plasma acceleration substantially influences the current channel width. The displacement of ions by the polarization field is given by $z_{dis}(t) = (\alpha/6)(Ze/m_i)(u/c)\dot{B}t^3$. In the collisionless case, its value becomes comparable with the depth of magnetic field penetration into the plasma at the time point $\tau_* = (4m_i/3Zm)^{1/2}\tau_{cl}$, and in the collisional case, at $\tau_* = (\pi m_i/2Zm)^{1/2}\tau_{cl}$. Because the accelerated mass thus increases, the axial plasma displacement can be more correctly estimated as $z_{dis}(t) = at^2/2$, where $a = \dot{B}/(12\pi m_i n/Z)^{1/2}$ is the acceleration [13]. The condition $ut = at^2/2$ sets the limit time τ_{lim} at which the field penetration velocity is still determined either by u_{cl} or by u_d. For collisionless plasma, this time is given by $\tau_{lim} = (8m_i/3Zm)^{1/2}\tau_{cl}$, and for collisional plasma, by $\tau_{lim} = (\pi m_i/Zm)^{1/2}\tau_{cl}$. The respective values of τ_{lim} and τ_* differ only by a factor of $\sqrt{2}$ as the cause of acceleration is independent of one or another approximation. The time τ_{lim} depends on the ratio $(m_i/Zm)^{1/2}$, and its value at any plasma density is determined only by the magnetic field rise rate. If $t \geq \tau_{lim}$, the magnetic field penetrates the plasma due to its aggregation.

Because of the plasma aggregation and simultaneous fast field penetration, the current channel width $\delta(t) = ut-at^2/2$ varies nonmonotonically during the current pulse. Its maximum $\delta(\tau_{lim}/2) = u\tau_{lim}/4$ falls on the time point $\tau_{lim}/2$, and on completion of conduction, the current channel width decreases to $\delta(t_c) = l[1-(\tau_c/\tau_{lim})]$.

For the Gamble I switch with a doubly ionized carbon plasma at $(m_i/Zm)^{1/2} \approx 105$, the limit time τ_{lim} is much longer than the conduction time: $\tau_{lim} \approx 125$ ns against $\tau_c \approx 50$ ns such that the maximum channel width can measure $\delta(\tau_{lim}/2) \approx 19$ cm. Because $\tau_c < \tau_{lim}/2$, the channel width at the end of conduction reaches only 18 cm, which is close to its experimental value $\sim l/2$ at an initial bridge length of 30 cm.

However, it is not the rule that the channel width should increase to $\sim l/2$. For example, in Gamble II experiments at $l = 10$ cm and same cathode radius (2.5 cm), the current rose to ~750 kA in ~50 ns. For such a current, not only the rate of its rise but also the plasma density should be increased, and this can increase the electron magnetization time to τ_d. If the Spitzer conductivity is $\sigma \approx 10^{14}$ s^{-1}, the magnetization of electrons is influenced by the rate of collisions at $n \approx 10^{15}$ cm^{-3}. The plasma density estimated by formula (1) is $n \approx 10^{14}$ cm^{-3}, which is an order of magnitude lower than the above value. Thus, the plasma in these experiments can also be taken collisionless. However, $\tau_{lim} \approx 65$ ns is only slightly greater than $\tau_c \approx 50$ ns. Likely, the current channel width first increases to ~3 cm in ~30 ns, and on completion of conduction, it decreases to ~2 cm.

Noteworthy is that the plasma displacement $z_{dis}(\tau_{dr})$ in the Gamble I switch is much smaller than the collisionless electron skin layer c/ω_{pe}, and in the Gamble II switch, its value is much greater than c/ω_{pe}. This suggests that the plasma in the Gamble II switch starts to be axially compressed very early in the current pulse. It is for this reason that the maximum channel width in the Gamble II switch is no greater than ~3 cm, whereas its value in the Gamble I switch reaches $\sim l/2$.

For a homogenous plasma bridge, the energy of ohmic heat is expended in increasing the thermal energy of electrons $N(t) = n\pi(r_a^2-r_c^2)z(t)$. Hence, the temperature of an electron during its drift is independent of the depth of magnetic field penetration and is given by $T_e(t) = \lambda B^2(r_c,t)/18\pi n$, where $\lambda = \ln(r_a/r_c)/[(r_a/r_c)^2-1]$ is a geometric factor. The increase in the temperature does not contradict the condition of drift velocity uniformity. The energy gained by electrons is expended in increasing the rate of their Larmor rotation. Their motion in an induced field is accelerated in the anode direction and decelerated in the opposite direction. The rate of electron rotation increases as $\propto B(r_c,t)$ and so does the electron cyclotron frequency. As a result, the Larmor radius during drift remains roughly constant.

For the Gamble I, Gamble II, and POP conditions, the time it takes for drift electrons to cross the interelectrode gap $r_a-r_c = 2.5$ cm measures respectively $\tau_{dr1} \approx 3$ ns ($n \approx 3\cdot10^{12}$ cm^{-3}, $l = 30$ cm), $\tau_{dr2} \approx 8$ ns ($n \approx 10^{14}$ cm^{-3}, $l = 10$ cm), and $\tau_{dr3} \approx 70$ ns ($n \approx 1.2\cdot10^{14}$ cm^{-3}, $l = 12$ cm). The maximum possible temperature in this case is no greater than $T_e(\tau_{dr1}) \approx 0.7$ keV ($\dot{B} = 0.32$ kGs/ns), $T_e(\tau_{dr2}) \approx 2.4$ keV ($\dot{B} = 1.2$ kGs/ns), and $T_e(\tau_{dr3}) \approx 50$ eV ($\dot{B} = 22$ Gs/ns). Clearly, such a temperature is gained only by a small group of electrons starting their drift near the cathode. With axial ion displacements early in the pulse, the energy balance changes in favor of increasing the kinetic energy, and this changes the energy distribution in the plasma [13].

5. Conclusion

The analysis shows that the conduction current behaves strictly as $I_c \propto n^{1/2}$ while varying the plasma density three orders of magnitude (from ~10^{11} cm^{-3} to ~10^{14} cm^{-3}) and the current rise rate an order of magnitude. Holding the conduction current constant at any bridge length requires that nl^2 be constant. These results agree with experimental data, suggesting that the concept of unipolar ion layer formation in a low-density plasma opening switch is justified.

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